

equations of parallel and perpendicular lines answer key

Equations of Parallel and Perpendicular Lines Answer Key

Understanding the equations of parallel and perpendicular lines is a fundamental aspect of coordinate geometry. These concepts not only help in solving mathematical problems but also play a crucial role in real-world applications, such as engineering, architecture, and computer graphics. This article will explore the properties of parallel and perpendicular lines, how to derive their equations, and provide an answer key to common questions and problems related to this topic.

Basic Concepts of Lines in Geometry

Before delving into the intricacies of parallel and perpendicular lines, it is essential to understand the foundational concepts of lines in a Cartesian coordinate system.

1. Slope of a Line

The slope of a line is a measure of its steepness and is calculated as the ratio of the vertical change (rise) to the horizontal change (run) between two points on the line. The formula for calculating the slope (m) between two points $((x_1, y_1))$ and $((x_2, y_2))$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

2. Slope-Intercept Form

The most common way to express the equation of a line is in the slope-intercept form:

$$y = mx + b$$

Where:

- (y) is the dependent variable,
- (m) is the slope of the line,
- (x) is the independent variable,
- (b) is the y-intercept (the value of (y) when $(x = 0)$).

Parallel Lines

Parallel lines are lines in the same plane that never intersect. They have the same slope but different y-intercepts.

1. Characteristics of Parallel Lines

- Same Slope: The slopes of parallel lines are equal. If line 1 has a slope (m_1) and line 2 has a slope (m_2) , then $(m_1 = m_2)$.
- Different Y-Intercepts: Although parallel lines have the same slope, their y-intercepts will differ, ensuring they do not meet.

2. Equation of a Parallel Line

To find the equation of a line parallel to a given line, you need the slope of the original line. For example, consider the line given by the equation:

$$y = 3x + 2$$

To find a line parallel to this line and passing through the point $((1, 4))$, follow these steps:

1. Identify the slope of the given line: $(m = 3)$.
2. Use the point-slope form of a line equation:

$$y - y_1 = m(x - x_1)$$

Substituting in the slope and the point:

$$y - 4 = 3(x - 1)$$

3. Simplifying yields:

$$y - 4 = 3x - 3$$
$$y = 3x + 1$$

Thus, the equation of the parallel line is $(y = 3x + 1)$.

Perpendicular Lines

Perpendicular lines intersect at a right angle (90 degrees). The slopes of two perpendicular lines are negative reciprocals of each other.

1. Characteristics of Perpendicular Lines

- Negative Reciprocals: If one line has a slope of (m_1) , the slope of a line that is perpendicular to it, (m_2) , will satisfy the equation:

$$m_1 \cdot m_2 = -1$$

Thus, $m_2 = -\frac{1}{m_1}$.

2. Equation of a Perpendicular Line

To find the equation of a line perpendicular to a given line, determine the negative reciprocal of the original line's slope. For example, consider the line given by the equation:

$$y = 2x + 3$$

To find a line perpendicular to this line and passing through the point $((2, 1))$:

1. Identify the slope of the given line: $(m = 2)$.
2. Find the slope of the perpendicular line:

$$m_{\text{perpendicular}} = -\frac{1}{2}$$

3. Use the point-slope form:

$$y - 1 = -\frac{1}{2}(x - 2)$$

4. Simplifying yields:

$$y - 1 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 2$$

Thus, the equation of the perpendicular line is $(y = -\frac{1}{2}x + 2)$.

Answer Key for Parallel and Perpendicular Lines

This section provides solutions to a series of problems related to parallel and perpendicular lines.

Problem Set

1. Find the equation of a line parallel to $(y = -4x + 5)$ that passes through the point $((3, 2))$.

- Solution: The slope is (-4) .

- Using point-slope form:

$$y - 2 = -4(x - 3)$$

$$y - 2 = -4x + 12$$

$$y = -4x + 14$$

2. Find the equation of a line perpendicular to $(y = \frac{1}{3}x - 2)$ that passes through the point $((6, 4))$.

- Solution: The slope is $(\frac{1}{3})$, so the perpendicular slope is (-3) .

- Using point-slope form:

$$y - 4 = -3(x - 6)$$

$$y - 4 = -3x + 18$$

$$y = -3x + 22$$

3. Determine if the lines $(3x - 4y = 12)$ and $(6x + 8y = 24)$ are parallel.

- Solution: Convert to slope-intercept form:

- Line 1: $(y = \frac{3}{4}x - 3)$ (slope = $(\frac{3}{4})$)

- Line 2: $(y = -\frac{3}{2}x + 3)$ (slope = $(-\frac{3}{2})$)

- Since the slopes are not equal, the lines are not parallel.

4. Find the intersection point of the lines $(y = 2x + 1)$ and $(y = -\frac{1}{2}x + 3)$.

- Solution: Set equations equal:

$$2x + 1 = -\frac{1}{2}x + 3$$

$$\frac{5}{2}x = 2 \rightarrow x = \frac{4}{5}$$

- Substitute back to find y :

$$y = 2\left(\frac{4}{5}\right) + 1 = \frac{13}{5}$$

- Intersection point is $\left(\frac{4}{5}, \frac{13}{5}\right)$.

Conclusion

Understanding the equations of parallel and perpendicular lines is essential for mastering coordinate geometry. By recognizing the relationship between slopes and utilizing the point-slope form, one can easily derive the equations of lines that meet these criteria. The skills developed through these concepts are invaluable not only in mathematics but also in various practical applications. The answer key provided here serves as a resource to verify understanding and solve problems related to parallel and perpendicular lines effectively.

Frequently Asked Questions

What is the slope of a line parallel to the line represented by the equation $y = 2x + 3$?

The slope of a line parallel to this line is also 2, since parallel lines have the same slope.

How do you find the equation of a line perpendicular to $y = -\frac{1}{3}x + 5$?

To find the equation of a perpendicular line, first determine the negative reciprocal of the slope. The slope of the given line is $-\frac{1}{3}$, so the perpendicular slope is 3. Using the point-slope form, you can write the equation as $y - y_1 = 3(x - x_1)$.

If two lines are parallel, what can you say about their slopes?

If two lines are parallel, their slopes are equal.

What is the relationship between the slopes of two

perpendicular lines?

The slopes of two perpendicular lines are negative reciprocals of each other. If one line has a slope m , the other line's slope will be $-1/m$.

How do you convert the equation of a line from standard form to slope-intercept form?

To convert from standard form $Ax + By = C$ to slope-intercept form $y = mx + b$, solve for y by isolating it on one side of the equation.

Given the line $3x + 4y = 12$, what is the slope of a line parallel to it?

First, convert the equation to slope-intercept form: $4y = -3x + 12$, which gives $y = -3/4x + 3$. The slope of a parallel line is $-3/4$.

If the slope of a line is 5, what is the slope of a line perpendicular to it?

The slope of a line perpendicular to a line with slope 5 is $-1/5$, which is the negative reciprocal of 5.

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